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POISSON-LIE T-DUALITY AND COMPLEX GEOMETRY IN N=2 SUPERCONFORMAL WZNW MODELS.

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Abstract

Poisson-Lie T-duality in N=2 superconformal WZNW models on the real Lie groups is considered. It is shown that Poisson-Lie T-duality is governed by the complexifications of the corresponding real groups endowed with Semenov-Tian-Shansky symplectic forms, i.e. Heisenberg doubles. Complex Heisenberg doubles are used to define on the group manifolds of the N=2 superconformal WZNW models the natural actions of the isotropic complex subgroups forming the doubles. It is proved that with respect to these actions N=2 superconformal WZNW models admit Poisson-Lie symmetries. Poisson-Lie T-duality transformation maps each model into itself but acts nontrivialy on the space of classical solutions.

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Introduction.

N=2 superconformal field theories (SCFT's) play the role of building blocks in the superstring vacua construction. The investigations in this direction was initiated in the pioneer works [1], [2], where it was shown that N=2 SCFT's may describe Calabi-Yau manifolds compactifications of superstrings. Since then the N=2 SCFT's, and the profound structures assotiated with them are an area of investigation. On the other hand, it is well known that the string vacua, considered as conformal field thoeries, in general, have deformations under which the geometry of of the target space changes. These deformations include the discrete duality transformations, so called T-duality, which are symmetries of the underlying conformal field theory [3], [4]. The well known example of T-duality is mirror symmetry in the Calaby-Yau manifolds compactifications of the superstring [5].

The Poisson-Lie (PL) T-duality, recently discoverd by C. Klimcik and P. Severa [6] is a generalization of the standard non-Abelian T-duality [7]- [11]. The main idea of the approach [6] is to replace the requirement of isometry of a σ-model with respect to some group by a weaker condition which is the Poisson-Lie symmetry of the theory. This generalized duality is associated with two groups forming a Drinfeld double [12] and the duality transformation exchanges their roles. This approach has recieved futher developments in the series of works [13], [14], [15], [16], [17].

In order to apply PL T-duality in superstring theory one needs to have the dual pairs of conformal and superconformal σ-models.

The simple example of dual pair of conformal σ-models associated with the $O(2,2)$ Drinfeld double was presented in work [18]. Then, it was shown in [14], [15] that WZNW models on the compact groups are the natural examples of PL dualizable σ-models.

The supersymmetric generalization of PL T-duality was considered in [19, 20, 21]. In particular, due to the close relation between N=2 superconformal WZNW (SWZNW) models and Drinfeld's double (Manin triple) structures on the corresponding group manifolds (Lie algebras) [22, 23], it was shown in [20] that N=2 SWZNW models possess very natural PL symmetry and PL T-dual σ-models for N=2 SWZNW models associated with real Drinfeld's doubles was constructed. Then the first example of PL T-duality in N=2 SWZNW models on the compact groups was obtained in [21] for $U(2)$-SWZNW model.

In the present paper we generalize the results of our preceding paper [21] to the case of N=2 superconformal WZNW models on the compact groups of higher dimensions. These models correspond to the complex Manin triples endowed with hermitian conjugation which conjugates isotropic subalgebras forming the Manin triples.

After a brief review of the classical N=1 superconformal WZNW models in the section 1, we describe in the section 2, a complex geometry of N=2 SWZNW models on the compact groups. We show that the Heisenberg double [24], [25] of the complexification of the N=2 SWZNW model group manifold plays the central role in PL T-duality: on the one hand, the Lagrangian of the model can be expressed (localy) in terms of the components of Semenov-Tian-Shansky symplectic form of the Heisenberg double, on the other hand, there is the natural action of the isotropic subgroups forming the double on the space of fields of the model. In the section 3 we show that each N=2 SWZNW model admits PL symmetries with respect to these actions which we use to show that PL T-duality transformation maps the model into itself but acts nontrivialy on the space of solutions. Though our results are concerned with N=2 SWZNW on the compact groups they can be straightforwardly generalized to the real noncompact groups.

1. The classical N=1 superconformal WZNW model.

In this section we briefly review the N=1 SWZNW models using superfield formalism [26].

We parametrize super world-sheet introducing the light cone coordinates $x_{\pm}$, and grassman coordinates $\Theta_{\pm}$. The generators of supersymmetry and covariant derivatives are

$$Q_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} + \imath \Theta_{\pm}\partial_{\mp},\ D_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} - \imath \Theta_{\pm}\partial_{\mp}. \tag{1}$$

They satisfy the relations

$$\{D_{\pm}, D_{\pm}\} = -\{Q_{\pm}, Q_{\pm}\} = -\imath 2\partial_{\pm},\ \{D_{\pm}, D_{\mp}\} = \{Q_{\pm}, Q_{\mp}\} = \{Q, D\} = 0, \tag{2}$$

where the brackets $\{,\}$ denote the anticommutator. The superfield of N=1 SWZNW model

$$G = g + \imath\Theta_-\psi_+ + \imath\Theta_+\psi_- + \imath\Theta_-\Theta_+F \tag{3}$$

takes values in a Lie group $\mathbf{G}$. We will assume that its Lie algebra $\mathbf{g}$ is endowed with ad-invariant nondegenerate inner product $<,>$.

The inverse group element G^{-1} is defined from the relation

$$G^{-1}G = 1 \tag{4}$$

and has the decomposition

$$G^{-1} = g^{-1} - \imath\Theta_-g^{-1}\psi_+g^{-1} - \imath\Theta_+g^{-1}\psi_-g^{-1} - \imath\Theta_-\Theta_+g^{-1}(F + \psi_-g^{-1}\psi_+ - \psi_+g^{-1}\psi_-)g^{-1} \tag{5}$$

The action of N=1 SWZNW model is given by

$$S_{swz} = \int d^2xd^2\Theta(< G^{-1}D_+G, G^{-1}D_-G >)$$
$$- \int d^2xd^2\Theta dt < G^{-1}\frac{\partial G}{\partial t}, \{G^{-1}D_-G, G^{-1}D_+G\} > . \tag{6}$$

The classical equations of motion can be obtained by making a variation of (6):

$$D_-(G^{-1}D_+G) = D_+(D_-GG^{-1}) = 0. \tag{7}$$

The action (6) is invariant under the super-Kac-Moody

$$\begin{aligned} \delta_{a_+}G(x_+, x_-, \Theta_+, \Theta_-) &= a_+(x_-, \Theta_+)G(x_+, x_-, \Theta_+, \Theta_-), \\ \delta_{a_-}G(x_+, x_-, \Theta_+, \Theta_-) &= -G(x_+, x_-, \Theta_+, \Theta_-)a_-(x_+, \Theta_-), \end{aligned} \tag{8}$$

where $a_\pm$ are $\mathbf{g}$-valued superfields and N=1 supersymmetry transformations [26]

$$\begin{aligned} G^{-1}\delta_{\epsilon_+}G &= (G^{-1}\epsilon_+Q_+G), \\ \delta_{\epsilon_-}GG^{-1} &= \epsilon_-Q_-GG^{-1}. \end{aligned} \tag{9}$$

In the following we will use supersymmetric version of Polyakov-Wiegman formula [27]

$$S_{swz}[GH] = S_{swz}[G] + S_{swz}[H] + \int d^2xd^2\Theta < G^{-1}D_+G, D_-HH^{-1} > . \tag{10}$$

It can be proved as in the non supersymmetric case.

2. Complex geometry in N=2 superconformal WZNW models.

In works [22, 23, 28] supersymmetric WZNW models which admit extended supersymmetry were studied and correspondence between extended supersymmetric WZNW models and finite-dimensional Manin triples was established in [22, 23]. By the definition [12], a Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ consists of a Lie algebra $\mathbf{g}$, with nondegenerate invariant inner product $<,>$ and isotropic Lie subalgebras $\mathbf{g}_\pm$ such that $\mathbf{g} = \mathbf{g}_+ \oplus \mathbf{g}_-$ as a vector space.

The corresponding Sugawara construction of N=2 Virasoro superalgebra generators was given in [22, 23, 28, 29].

To make a connection between Manin triple construction of [22, 23] and approach of [28] based on the complex structures on Lie algebras the following comment is relevant.

Let $\mathbf{g}$ be a real Lie algebra and J be a complex structure on the vector space $\mathbf{g}$. J is referred to as the complex structure on the Lie algebra $\mathbf{g}$ if J satisfies the equation

$$[Jx, Jy] - J[Jx, y] - J[x, Jy] = [x, y] \tag{11}$$

for any elements x, y from $\mathbf{g}$. It is clear that the corresponding Lie group is a complex manifold with left (or right) invariant complex structure. In the following we will denote the real Lie group and the real Lie algebra with the complex structure satisfying (11) as the pairs $(\mathbf{G}, J)$ and $(\mathbf{g}, J)$ correspondingly.

Suppose the existence of the nondegenerate invariant inner product $<,>$ on $\mathbf{g}$ so that the complex structure J is skew-symmetric with respect to $<,>$. In this case it is not difficult to establish the correspondence between complex Manin triples and complex structures on the Lie algebras. Namely, for each complex Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ exists the canonic complex structure on the Lie algebra $\mathbf{g}$ such that subalgebras $\mathbf{g}_\pm$ are its $\pm\imath$ eigenspaces. On the other hand, for each real Lie algebra $\mathbf{g}$ with nondegenerate invariant inner product and skew-symmetric complex structure J on this algebra one can consider the complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$. Let $\mathbf{g}_\pm$ be $\pm\imath$ eigenspaces of J in the algebra $\mathbf{g}^{\mathbb{C}}$ then $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_+, \mathbf{g}_-)$ is a complex Manin triple . Moreover it can be proved that there exists the one-to-one correspondence between the complex Manin triple endowed with antilinear involution which conjugates isotropic subalgebras $\tau : \mathbf{g}_\pm \to \mathbf{g}_\mp$ and the real Lie algebra endowed with ad-invariant nondegenerate inner product $<,>$ and the complex structure J which is skew-symmetric with respect to $<,>$ [22]. Therefore we can use this conjugation to extract the real form from the complex Manin triple.

If the complex structure J on the Lie algebra is fixed then it defines the second supersymmetry transformation [28]

$$\begin{aligned}
(G^{-1}\delta_{\eta_+}G)^a &= \eta_+(J_l)^a_b(G^{-1}D_+G)^b, \\
(\delta_{\eta_-}GG^{-1})^a &= \eta_-(J_r)^a_b(D_-GG^{-1})^b,
\end{aligned} \tag{12}$$

where J_l, J_r are the left invariant and right invariant complex structures on $\mathbf{G}$ which correspond to the complex structure J.

To specify our presentation we concentrate in this paper on N=2 SWZNW models on the compact groups (the extension on the noncompact groups is straightforward) that is we shall consider complex Manin triples such that the corresponding antilinaer involutions will coincide with the hermitian conjugations. Hence it will be implied in the following that $\mathbf{G}$ is a subgroup in the group of unitary matrices and the matrix elements of the superfield G satisfy the relations:

$$\bar{g}^{mn} = (g^{-1})^{nm},\ \bar{\psi}^{mn}_\pm = (\psi^{-1})^{nm}_\pm,\ \bar{F}^{mn} = (F^{-1})^{nm}, \tag{13}$$

where we have used the following notations

$$\psi^{-1}_\pm = -g^{-1}\psi_\pm g^{-1},\ F^{-1} = -g^{-1}(F + \psi_- g^{-1}\psi_+ - \psi_+ g^{-1}\psi_-)g^{-1}. \tag{14}$$

Now we have to consider some geometric properties of the N=2 SWZNW models closely related with the existence of the complex structures on the corresponding groups.

Let's fix some compact Lie group with the left invariant complex structure $(\mathbf{G}, J)$ and consider its Lie algebra with the complex structure $(\mathbf{g}, J)$. The complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$ has the Manin triple structure $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_+, \mathbf{g}_-)$. The Lie group version of this triple is the double Lie group $(\mathbf{G}^{\mathbb{C}}, \mathbf{G}_+, \mathbf{G}_-)$ [24, 30, 25], where the exponential subgroups $\mathbf{G}_\pm$ correspond to the Lie algebras $\mathbf{g}_\pm$. The real Lie group $\mathbf{G}$ is extracted from its complexification with help of the hermitian conjugation τ

$$\mathbf{G} = \{g \in \mathbf{G}^{\mathbb{C}} | \tau(g) = g^{-1}\} \tag{15}$$

Each element $g \in \mathbf{G}^{\mathbb{C}}$ from the vicinity $\mathbf{G_1}$ of the unit element from $\mathbf{G}^{\mathbb{C}}$ admits two decompositions

$$g = g_+ g_-^{-1} = \tilde{g}_- \tilde{g}_+^{-1}, \tag{16}$$

where $\tilde{g}_\pm$ are dressing transformed elements of $g_\pm$ [30]:

$$\tilde{g}_\pm = (g_\pm^{-1})^{g_\mp} \tag{17}$$

Taking into account (15) and (16) we conclude that the element g ($g \in \mathbf{G_1}$) belongs to $\mathbf{G}$ iff

$$\tau(g_\pm) = \tilde{g}_\mp^{-1} \tag{18}$$

These equations mean that we can parametrize the elements from

$$\mathbf{C_1} \equiv \mathbf{G_1} \cap \mathbf{G} \tag{19}$$

by the elements from the complex group $\mathbf{G}_+$ (or $\mathbf{G}_-$), i.e. we can introduce complex coordinates (they are just matrix elements of g_+ (or g_-)) in the cell $\mathbf{C_1}$. To do it one needs to solve with respect to g_- the equation:

$$\tau(g_-) = (g_+)^{g_-^{-1}} \tag{20}$$

(to introduce $\mathbf{G}_-$-coordinates on $\mathbf{G_1}$ one needs to solve with respect to g_+ the equation

$$\tau(g_+) = (g_-)^{g_+^{-1}}). \tag{21}$$

Thus the formulas (16), (20) ((21)) define the map

$$\phi_1^+ : \mathbf{G}_+ \rightarrow \mathbf{C_1} \tag{22}$$

$$(\phi_1^- : \mathbf{G}_- \rightarrow \mathbf{C_1}) \tag{23}$$

For the N=2 SWZNW model on the group $\mathbf{G}$ we obtain from (16) the decompositions for the superfield (4) (which takes values in $\mathbf{C_1}$)

$$G(x_+, x_-) = G_+(x_+, x_-) G_-^{-1}(x_+, x_-) = \tilde{G}_-(x_+, x_-) \tilde{G}_+^{-1}(x_+, x_-) \tag{24}$$

Due to (24), (10) and the definition of Manin triple we can rewrite the action (6) for this superfield in the manifestly real form

$$S_{swz} = -\frac{1}{2} \int d^2x d^2\Theta (< \rho_+^+, \rho_-^- > + < \tilde{\rho}_+^-, \tilde{\rho}_-^+ >), \tag{25}$$

where the superfields

$$\rho^{\pm} = G_{\pm}^{-1} D G_{\pm},\ \tilde{\rho}^{\pm} = \tilde{G}_{\pm}^{-1} D \tilde{G}_{\pm} \tag{26}$$

correspond to the left invariant 1-forms on $\mathbf{G}_{\pm}$

$$r^{\pm} = g_{\pm}^{-1} d g_{\pm},\ \tilde{r}^{\pm} = \tilde{g}_{\pm}^{-1} d \tilde{g}_{\pm}. \tag{27}$$

To proceed futher we need to proove the following

LEMMA 1.

1) $\mathbf{g}_{+}$-valued 1-form r^{+} considering as a form on $(\mathbf{C_1}, J)$ is holomorphic.

2) $\mathbf{g}_{-}$-valued 1-form $\tilde{r}^{-}$ considering as a form on $(\mathbf{C_1}, J)$ is antiholomorphic.

Proof.

We identify the Lie algebra $\mathbf{g}^{\mathbb{C}}$ with the space of complex left invariant vector fields on the group $\mathbf{G}$. Let

$$\{R_i, i = 1, ..., d\}, \tag{28}$$

be the basis in the Lie subalgebra $\mathbf{g}_{+}$ and

$$\{R^i, i = 1, ..., d\}, \tag{29}$$

be the basis in the Lie subalgebra $\mathbf{g}_{-}$ so that (28, 29) constitute the orthonormal basis in $\mathbf{g}^{\mathbb{C}}$:

$$< R^i, R_j >= \delta^i_j. \tag{30}$$

Denote by ξ the canonic $\mathbf{g}^{\mathbb{C}}$-valued left invariant 1-form on the group $\mathbf{G}$. It is obvious that

$$\begin{gathered} \xi = \xi^j R_j + \xi_j R^j, \\ J\xi^k = -\imath \xi^k,\ J\xi_k = \imath \xi_k. \end{gathered} \tag{31}$$

We can write also

$$\begin{aligned} r^{+} = r^i R_i,\ r^{-} = r_i R^i, \\ \tilde{r}^{+} = \tilde{r}^i R_i,\ \tilde{r}^{-} = \tilde{r}_i R^i, \\ l^{+} \equiv d g_{+} g_{+}^{-1} = l^i R_i,\ l^{-} \equiv d g_{-} g_{-}^{-1} = l_i R^i, \\ \tilde{l}^{+} \equiv d \tilde{g}_{+} \tilde{g}_{+}^{-1} = \tilde{l}^i R_i,\ \tilde{l}^{-} \equiv d \tilde{g}_{-} \tilde{g}_{-}^{-1} = \tilde{l}_i R^i. \end{aligned} \tag{32}$$

1) Using the first decomposition from (16) we get

$$\xi = g_{-} r^{+} g_{-}^{-1} - l^{-}. \tag{33}$$

Let's introduce the matrices

$$\begin{aligned} g_{-} R_i g_{-}^{-1} = M_{ij} R^j + N_i^j R_j, \\ g_{+} R^i g_{+}^{-1} = P^{ij} R_j + Q^i_j R^j, \\ g_{-} R^i g_{-}^{-1} = (N^*)^i_j R^j, \\ g_{+} R_i g_{+}^{-1} = (Q^*)_i^j R_j. \end{aligned} \tag{34}$$

Using these matrices and (30) we can express the 1-forms r^i, r_i, l^i, l_i in terms of ξ^i:

$$\begin{aligned} r^j &= (N^{-1})^j_i \xi^i, \\ l_i &= -\xi_i + M_{ji}(N^{-1})^j_k \xi^k, \\ r_i &= ((N^*)^{-1})^j_i(-\xi_j + M_{nj}(N^{-1})^n_k \xi^k), \\ l^i &= (N^{-1})^j_k (Q^*)^i_j \xi^k. \end{aligned} \tag{35}$$

Taking into account (31, 35) we obtain

$$Jr^k = -\imath r^k, \ \ Jl^k = -\imath l^k. \tag{36}$$

Thus $r^k, l^k, k = 1, ..., d$ are (1,0)-forms and its complex conjugated $\bar{r}^k, \bar{l}^k, k = 1, ..., d$ are (0,1)-forms on $(\mathbf{C_1}, J)$. Because the forms $r^k, \bar{r}^k$ are linear independent on $(\mathbf{C_1}, J)$ and they are linear independent on $\mathbf{G}_+$ we conclude that the map ϕ_1 is holomorphic [31] , where the complex structure on $\mathbf{G}_+$ is given by the multiplication by $\imath$. Since the forms r^k are holomorphic as the forms on $\mathbf{G}_+$ we obtain the statement 1 of the lemma.

2) Using the second decomposition from (16) we can write

$$\xi = \tilde{g}_+ \tilde{r}^- \tilde{g}_+^{-1} - \tilde{l}^+. \tag{37}$$

Introducing the matrices

$$\begin{aligned} \tilde{g}_- R_i \tilde{g}_-^{-1} &= \tilde{M}_{ij} R^j + \tilde{N}^j_i R_j, \\ \tilde{g}_+ R^i \tilde{g}_+^{-1} &= \tilde{P}^{ij} R_j + \tilde{Q}^i_j R^j, \\ \tilde{g}_- R^i \tilde{g}_-^{-1} &= (\tilde{N}^*)^i_j R^j, \\ \tilde{g}_+ R_i \tilde{g}_+^{-1} &= (\tilde{Q}^*)^j_i R_j. \end{aligned} \tag{38}$$

and using (30) one can express the 1-forms $\tilde{r}^i, \tilde{r}_i, \tilde{l}^i, \tilde{l}_i$ in terms of ξ^i:

$$\begin{aligned} \tilde{r}_j &= (\tilde{Q}^{-1})^i_j \xi_i, \\ \tilde{l}^i &= -\xi^i + \tilde{P}^{ji}(\tilde{Q}^{-1})^k_j \xi_k, \\ \tilde{r}^i &= ((\tilde{Q}^*)^{-1})^i_j(-\xi^j + \tilde{P}^{nj}(\tilde{Q}^{-1})^k_n \xi_k), \\ \tilde{l}_i &= (Q^{-1})^k_j (N^*)^j_i \xi_k. \end{aligned} \tag{39}$$

In view of (31, 39) we obtain

$$J\tilde{r}_k = \imath \tilde{r}_k, \ \ J\tilde{l}_k = \imath \tilde{l}_k. \tag{40}$$

Thus $\tilde{r}_k, \tilde{l}_k, k = 1, ..., d$ are (0,1)-forms and its complex conjugated $\bar{\tilde{r}}_k, \bar{\tilde{l}}_k, k = 1, ..., d$ are (1,0)-forms on $(\mathbf{C_1}, J)$. Because the forms $\tilde{r}_k, \bar{\tilde{r}}_k$ are linear independent on $(\mathbf{C_1}, J)$ and they are linear independent on $\mathbf{G}_-$ we conclude that the map ϕ_1^- is holomorphic, where the complex structure on $\mathbf{G}_-$ is given by the multiplication by $-\imath$. Since the forms $\tilde{r}_k$ are antiholomorphic as the forms on $\mathbf{G}_-$ we obtain the statement 2 of the lemma.

Due to Lemma 1 the forms

$$\{r^i, \bar{r}^i\}, i = 1, ..., d \tag{41}$$

constitute the basis of holomorphic and antiholomorphic 1-forms in the open subset $\mathbf{C_1}$. Consequently, in the basis of (28, 29) we can write the components of $\mathbf{g}_{\pm}$-valued 1-forms $\rho^-, \tilde{\rho}^+$ as follows

$$\rho_i = E_{i\bar{j}}\bar{\rho}^j + E_{ij}\rho^j,$$
$$\tilde{\rho}^i = \bar{\rho}_i = E_{\bar{j}i}\rho^j + E_{\bar{i}\bar{j}}\bar{\rho}^j, \tag{42}$$

where we have used the notations

$$\bar{E}_{i\bar{j}} = E_{\bar{j}i}, \;\; \bar{E}_{ij} = E_{\bar{i}\bar{j}}. \tag{43}$$

Thus the Lagrangian of the action (25) will has the following form

$$\Lambda = \rho^i_+(\rho_-)_i - \tilde{\rho}^i_-(\tilde{\rho}_+)_i =$$
$$\frac{1}{2}((E_{ij} - E_{ji}) + (E_{ij} + E_{ji}))\rho^i_+\rho^j_- + E_{i\bar{j}}(\rho^i_+\bar{\rho}^j_- - \rho^i_-\bar{\rho}^j_+) +$$
$$\frac{1}{2}((E_{\bar{i}\bar{j}} - E_{\bar{j}\bar{i}}) + (E_{\bar{i}\bar{j}} + E_{\bar{j}\bar{i}}))\bar{\rho}^i_+\bar{\rho}^j_-. \tag{44}$$

Let's show that

$$E_{ij} + E_{ji} = E_{\bar{i}\bar{j}} + E_{\bar{j}\bar{i}} = 0. \tag{45}$$

Using (16) we can represent the kinetic term of the action (6) in the following form

$$-2 < G^{-1}D_+G, G^{-1}D_-G >= (E_{ij}+E_{ji})\rho^i_+\rho^j_- + E_{i\bar{j}}(\rho^i_+\bar{\rho}^j_- - \rho^i_-\bar{\rho}^j_+) + (E_{\bar{i}\bar{j}}+E_{\bar{j}\bar{i}})\bar{\rho}^i_+\bar{\rho}^j_-. \tag{46}$$

This expression means that the bilinear form $<,>$ on $\mathbf{G}$ written in the basis (41) has the following nonzero components:

$$-K_{ij} = E_{ij} + E_{ji}, \;\; -K_{i\bar{j}} = 2E_{i\bar{j}}, \;\; -K_{\bar{i}\bar{j}} = E_{\bar{i}\bar{j}} + E_{\bar{j}\bar{i}}. \tag{47}$$

From the other hand the complex structure J is skew-symmetric with respect to $<,>$, therefore (45) should be satisfied.

LEMMA 2.

The Lagrangian Λ can be expressed in terms of Semenov-Tian-Shansky symplectic form Ω:

$$\Lambda = \frac{\imath}{2}\Omega_{cb}J^c_a\rho^a_+\rho^b_-, \tag{48}$$

where we have used common notation $\rho^a, a = 1, ..., 2d$ for the 1-forms $\rho^i, \bar{\rho}^i$.

Poof.

In the open subset $\mathbf{C_1}$, where the decomposition (16) takes place, Semenov-Tian-Shansky simplectic form Ω can be represented as follows [25, 16]

$$\Omega = r^i \wedge r_i + \tilde{r}^i \wedge \tilde{r}_i. \tag{49}$$

Due to the forms $r_i, \tilde{r}^i$ can be expressed in terms of $r^i, \bar{r}^i$ in perfect analogy to $\rho_i, \tilde{\rho}^i$ the restriction of Ω on any world-sheet (which should not be confused with the super world-sheet of N=2 SWZNW model under consideration) is given by

$$\Omega\mid_{\Sigma} = dx_+ \wedge dx_-(\frac{1}{2}(E_{ij} - E_{ji})(r^i_+r^j_- - r^i_-r^j_+) + 2E_{i\bar{j}}(r^i_+\bar{r}^j_- - r^i_-\bar{r}^j_+) -$$
$$\frac{1}{2}(E_{\bar{i}\bar{j}} - E_{\bar{j}\bar{i}})(\bar{r}^i_+\bar{r}^j_- - \bar{r}^i_-\bar{r}^j_+)) \tag{50}$$

Comparing (44) with (50) and taking into account (45) and Lemma 1 we obtain (48).

To generalize (16), (18) one have to consider the set W (which we shall assume in the following to be discret and finite set) of classes $\mathbf{G}_+\backslash\mathbf{G}^{\mathbb{C}}/\mathbf{G}_-$ and pick up a representative w for each class $[w] \in W$. It gives us the stratification of $\mathbf{G}^{\mathbb{C}}$ [25]:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W} \mathbf{G}_+ w \mathbf{G}_- = \bigcup_{[w]\in W} \mathbf{G}_{\mathbf{w}} \tag{51}$$

There is the second stratification:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W} \mathbf{G}_- w \mathbf{G}_+ = \bigcup_{[w]\in W} \mathbf{G}^{\mathbf{w}} \tag{52}$$

We shall assume, in the following, that the representatives w have picked up to satisfy the unitarity condition:

$$\tau(w) = w^{-1} \tag{53}$$

It allows us to generalize (16), (18) as follows

$$g = g_+ w g_-^{-1} = \tilde{g}_- w \tilde{g}_+^{-1}. \tag{54}$$

It will be more convenient to rewrite this decomposition in another form

$$g = w g_+ g_-^{-1} = w \tilde{g}_- \tilde{g}_+^{-1}. \tag{55}$$

The group elements $g_\pm, \tilde{g}_\pm$ from this formula should not be confused with the group elements from (54) (of couse they are mutualy related but not coincide). To make the decompositions (55) unumbigously determined we should demand that

$$g_+ \in \mathbf{G}_+^{\mathbf{w}},\ \ \tilde{g}_- \in \mathbf{G}_-^{\mathbf{w}}, \tag{56}$$

where

$$\mathbf{G}_+^{\mathbf{w}} = \mathbf{G}_+ \cap w^{-1}\mathbf{G}_+ w,\ \ \mathbf{G}_-^{\mathbf{w}} = \mathbf{G}_- \cap w^{-1}\mathbf{G}_- w. \tag{57}$$

The helpful example, where the formulas (51- 53, 55) take place is the Bruhat decomposition of the complexification of an even-dimensional semisimple compact Lie group with an appropriate maximal torus decomposition.

In order to the element g belongs to the real group $\mathbf{G}$ the elements $g_\pm, \tilde{g}_\pm$ from (55) should satisfy (18). Thus the formulas (55, 56), (20) ((21)) define the map

$$\phi_w^+ : \mathbf{G}_+^{\mathbf{w}} \to \mathbf{C}_{\mathbf{w}} \equiv \mathbf{G}_{\mathbf{w}} \cap \mathbf{G} \tag{58}$$

$$(\phi_w^- : \mathbf{G}_-^{\mathbf{w}} \to \mathbf{C}_{\mathbf{w}} \equiv \mathbf{G}_{\mathbf{w}} \cap \mathbf{G}). \tag{59}$$

We can obtain the corresponding generalization of Lemma 1 for the $\mathbf{g}_+^{\mathbf{w}}$-components of the form r^+, where $\mathbf{g}_+^{\mathbf{w}} = \mathbf{g}_+ \cap w^{-1}\mathbf{g}_+ w$ ($\mathbf{g}_-^{\mathbf{w}}$-components of the form $\tilde{r}^-$, where $\mathbf{g}_-^{\mathbf{w}} = \mathbf{g}_- \cap w^{-1}\mathbf{g}_- w$) proving by analogy with the proof of the Lemma 1 that the map (58) ((59)) is holomorphic.

Using the appropriate generalization of (24) and taking into account (56, 57) we can conclude that the action for the map into the cell $\mathbf{C}_{\mathbf{w}}$ is given by the formula (25). Due

to the map (58) is holomorphic the same is true for the Lagrangian: it is given by (48), where $\rho^a R_a \in \mathbf{g}_+^{\mathbf{w}}$.

It is clear that the formula (25) is correct inside the super world-sheet domain where the superfields take values in the cell $\mathbf{C_w}$. On the boundaries of these domains, where the jumps from one cell to another one is appeared some additional terms should be added, but as it will be explained below, for our purposes it will suffice to ignore these terms.

The formulas (20), (55), (25) mean that there is a natural action of the complex group $\mathbf{G}_+$ on $\mathbf{G}$, and the set W parametrizes $\mathbf{G}_+$-orbits $\mathbf{C_w}$. It's obvious that there is also the action of the complex group $\mathbf{G}_-$ on $\mathbf{G}$ so that the analogies of the formulas (44, 45, 48) can be obtained by the similar way.

3. Poisson-Lie symmetry in N=2 SWZNW model.

In view of (20), (55), (25) we can consider the $(\mathbf{G}, J)$-SWZNW model as a σ-model on the orbits of the complex Lie group $\mathbf{G}_+$ and find the equations of motion making a variation of the action (25) under the right action of this group on itself. We will consider the variations which are non zero only inside the super world-sheet domains, where the jumps from one cell $\mathbf{C_w}$ to another one is appeared. Thus we can take not into account the corresponding boundary terms omissioned in the action (25).

The set of holomorphic maps $\{\phi_w^+,\ w \in W\}$ define the action of $\mathbf{G}_+$ on the group $\mathbf{G}$:

$$h_+ \cdot g \equiv w g_+ h_+ g_-^{-1}(g_+ h_+),\ h_+ \in \mathbf{G}_+, \tag{60}$$

where $g_-^{-1}(g_+h_+)$ means the solution of (20) with the argument g_+h_+. The vector fields $\{S_i, i = 1, ..., d\}$ generating this action are the holomorphic vector fields on the cells $\mathbf{C_w}$.

Remark that in the case when $\mathbf{G}$ is an even-dimensional semisimple compact Lie group and (51- 53, 55) are given by the Bruhat decomposition, these vector fields will coincide with the classical screening currents in the Wakimoto representations of $\hat{\mathbf{g}}$ [32]. This is due to the fact that the classical screening currents in the Wakimoto representations are given by the right action of the maximal nilpotent subgroup $\mathbf{N}_+$ on the big cell of the corresponding flag manifold $\mathbf{G}/\mathbf{B}_-$, where $\mathbf{B}_-$ is the Borelian subgroup of the group $\mathbf{G}$ ($\mathbf{N}_+ \in \mathbf{B}_+$) [33].

Let's consider a variation of (25) under the vector field $Z = Z^i S_i + Z^{\bar{i}} \bar{S}_i$ for the map into the cell $\mathbf{C_1}$. We obtain on the extremals

$$\begin{aligned} &D_+(A_-)_i + D_-(A_+)_i - L_{S_i}\Lambda = 0, \\ &D_+(A_-)_{\bar{i}} + D_+(A_+)_{\bar{i}} - L_{\bar{S}_i}\Lambda = 0, \end{aligned} \tag{61}$$

where $L_{S_i}, L_{\bar{S}_i}$ mean the Lie derivatives along the vector fields $S_i, \bar{S}_i$ and the Noether currents $A_i, A_{\bar{i}}$ are given by

$$\begin{aligned} (A_-)_i &= E_{i\bar{j}}\bar{\rho}_-^j + E_{ij}\rho_-^j, \\ (A_+)_i &= -E_{i\bar{j}}\bar{\rho}_+^j - E_{ji}\rho_+^j, \\ (\bar{A}_\pm)_i &= (A_\pm)_{\bar{i}}. \end{aligned} \tag{62}$$

Due to the fields S_i are holomorphic we get from (48)

$$L_{S_i}\Lambda_{ab} = \frac{\imath}{2} J_a^c L_{S_i}\Omega_{cb}. \tag{63}$$

Now one have to find the Lie derivative of the Semenov-Tian-Shansky simplectic form. Because $L_{S_j} = i_{S_j} d + d i_{S_j}$ and Ω is closed, we have

$$L_{S_i}\Omega = d i_{S_j}\Omega = 2dr_j,\ \ L_{\bar{S}_i}\Omega = d i_{\bar{S}_j}\Omega = 2d\bar{r}_j, \tag{64}$$

where we have used (49). In view of (27) $dr_j, d\bar{r}_j$ can be expressed in terms of $r_j, \bar{r}_j$ again

$$\begin{aligned} dr_j &= -\frac{1}{2} f_j^{ik} r_i \wedge r_k, \\ d\bar{r}_j &= -\frac{1}{2} \bar{f}_j^{ik} \bar{r}_i \wedge \bar{r}_k, \end{aligned} \tag{65}$$

where f_j^{ik} are the structure constants of the Lie algebra $\mathbf{g}_-$. Thus the action (60) is Poisson action [24] with respect to the Poisson structure defined by the symplectic form Ω. Therefore

$$\begin{aligned} L_{S_i}\Lambda &= \imath f_j^{ik} (J\rho_+)_i (\rho_-)_k, \\ L_{\bar{S}_i}\Lambda &= \imath \bar{f}_j^{ik} (J\bar{\rho}_+)_i (\bar{\rho}_-)_k. \end{aligned} \tag{66}$$

Taking into account the relations

$$(A_-)_i = (\rho_-)_i,\ \ (A_+)_i = \imath (J\rho_+)_i, \tag{67}$$

where the second equality is due to (62) and Lemma 1, we get that the following PL symmetry conditions are satisfied on the extremals of N=2 $(\mathbf{G}, J)$-SWZNW model

$$\begin{aligned} L_{S_i}\Lambda &= f_i^{jk} (A_+)_j (A_-)_k \\ L_{\bar{S}_i}\Lambda &= \bar{f}_i^{jk} (A_+)_{\bar{j}} (A_-)_{\bar{k}}. \end{aligned} \tag{68}$$

By demanding the closure of (68): $[L_{S_i}, L_{S_j}] = f_{ij}^k L_{S_k}$, we shall have the consistency condition

$$f_{ij}^n f_n^{km} = f_j^{nm} f_{in}^k - f_j^{nk} f_{in}^m \ - f_i^{nm} f_{jn}^k + f_i^{nk} f_{jn}^m, \tag{69}$$

which is satisfied due to the Jacoby identity in the Lie algebra $\mathbf{g}^{\mathbb{C}}$.

As it is easy to see from (61) the eq. (68) are equivalent to zero curvature equations for the F_{+-}-component of the super stress tensor F_{MN}

$$\begin{aligned} (F_{+-})_i &\equiv D_+(A_-)_i + D_-(A_+)_i - f_i^{nm} (A_+)_n (A_-)_m = 0 \\ (F_{+-})_{\bar{i}} &\equiv D_+(A_-)_{\bar{i}} + D_-(A_+)_{\bar{i}} - \bar{f}_i^{nm} (A_+)_{\bar{n}} (A_-)_{\bar{m}} = 0 \end{aligned} \tag{70}$$

Using the standard arguments of the super Lax construction [34] one can show that from (67) it follows that the connection is flat

$$F_{MN} = 0,\ \ M, N = (+, -, +, -). \tag{71}$$

The equations (70) are the supersymmetric generalization of Poisson-Lie symmetry conditions from the work [6]. Indeed, the Noether currents $A_i, A_{\bar{i}}$ are generators of $\mathbf{g}_+$- action, while the structure constants in (70) correspond to the Lie algebra $\mathbf{g}_-$ which is Drinfeld's dual to $\mathbf{g}_+$ [35].

Now we turn to the maps into the remainder cells $\mathbf{C_w}, w \in W$. For each $w \in W$ the action and the Lagrangian for the map into $\mathbf{C_w}$ are given by (25) and (48) with the restriction (56). Consequently, the corresponding equations of motion will coincide with (70), where only $\mathbf{g^w}_-$- components of the Noether currents (62) are not identicaly zeroes.

Thus we have shown that N=2 $(\mathbf{G}, J)$-SWZNW model admits Poisson-Lie symmetry with respect to the complex group $\mathbf{G}_+$.

4. Poisson-Lie T-self-duality of N=2 SWZNW models.

The PL T-dual to $(\mathbf{G}, J)$-SWZNW σ-model should obey the conditions as (70) but with the roles of the Lie algebras $\mathbf{g}_\pm$ interchanged [6].

To find the action of this model we start from the maps into the cell $\mathbf{C_1}$. Due to (71) we may associate to each extremal surface $G_+(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}_+$, a map ("Noether charge") $V_-(x_+, x_-, \Theta_+, \Theta_-)$ from the super world-sheet into the group $\mathbf{G}_-$ such that

$$(A_\pm)_i = -(D_\pm V_- V_-^{-1})_i. \tag{72}$$

Now we build the following surface in the double $\mathbf{G}^{\mathbb{C}}$:

$$F(x_+, x_-, \Theta_+, \Theta_-) = G_+(x_+, x_-, \Theta_+, \Theta_-) V_-(x_+, x_-, \Theta_+, \Theta_-). \tag{73}$$

In view of (67) it is natural to represent V_- as the product

$$V_- = G_-^{-1} H_-^{-1} \tag{74}$$

, where G_- is determined from (20) and H_- satisfy the equation

$$D_- H_- = 0. \tag{75}$$

Therefore the surface (73) can be rewritten in the form

$$F(x_\pm, \Theta_\pm) = G_(x_\pm, \Theta_\pm) H_-^{-1}(x_+, \Theta_-), \tag{76}$$

where $G_(x_\pm, \Theta_\pm) \in \mathbf{C_1}$ is the solution of $(\mathbf{G})$-SWZNW model restricted to the corresponding domain of the super world-sheet.

The solution and the "Noether charge" of the dual σ-model are given by "dual" parametrization of the surface (73) [6]

$$F(x_+, x_-, \Theta_+, \Theta_-) = \breve{G}_-(x_+, x_-, \Theta_+, \Theta_-) \breve{V}_+(x_+, x_-, \Theta_+, \Theta_-), \tag{77}$$

where $\breve{G}_-(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}_-$ and $\breve{V}_+(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}_+$. Thus in the dual σ-model Drinfeld's dual group to the group $\mathbf{G}_+$should acts, i.e. it should be a σ-model on the orbits of the group $\mathbf{G}_-$ and with respect to this action the dual to (68) PL symmetry conditions should be satisfied:

$$\begin{aligned} L_{S^i} \breve{\Lambda} &= f^i_{jk} (\breve{A}_+)^j (\breve{A}_-)^k, \\ L_{\bar{S}^i} \breve{\Lambda} &= \bar{f}^i_{jk} (\breve{A}_+)^{\bar{j}} (\breve{A}_-)^{\bar{k}}, \end{aligned} \tag{78}$$

where $\{S^i, \bar{S}^i, i = 1, ..., d\}$ are the vector fields which generate the $\mathbf{G}_-$-action, $\breve{\Lambda}, \breve{A}^j_\pm, \breve{A}^{\bar{j}}_\pm$ are the Lagrangian and the Noether currents in the dual σ-model. Taking into account

the second decomposition from (16), and holomorphicity of ϕ_1^- it is easy to see that the right action of $\mathbf{G}_-$ on itself defines the action on the cell $\mathbf{C_1}$:

$$h_- \cdot g \equiv \tilde{g}_- h_- \tilde{g}_+^{-1}(\tilde{g}_- h_-),\ h_- \in \mathbf{G}_-, \tag{79}$$

so that the vector fields generating this action are the holomorphic vector fields. Therefore, due to the formula (49) the Lagrangian Λ of the initial N=2 SWZNW model on the cell $\mathbf{C_1}$ written in $\mathbf{G}_-$-coordinates satisfy (78). Thus the dual σ-model on the cell $\mathbf{C_1}$ is governed by the action (25) and we can rewrite the right hand side of (77) analogously to (76):

$$F(x_\pm, \Theta_\pm) = \breve{G}(x_\pm, \Theta_\pm)\breve{H}_+^{-1}(x_+, \Theta_-), \tag{80}$$

where $\breve{G}(x_\pm, \Theta_\pm) \in \mathbf{C_1}$ and

$$D_- \breve{H}_+ = 0. \tag{81}$$

For the maps into the remainder cells $\mathbf{C_w}$ we have also the surfaces in the double $\mathbf{G}^{\mathbb{C}}$ written in two ways:

$$\begin{aligned} F(x_+, x_-, \Theta_+, \Theta_-) &= wG_+(x_+, x_-, \Theta_+, \Theta_-)V_-(x_+, x_-, \Theta_+, \Theta_-) \\ &= w\breve{G}_-(x_+, x_-, \Theta_+, \Theta_-)\breve{V}_+(x_+, x_-, \Theta_+, \Theta_-), \end{aligned} \tag{82}$$

where $G_+(x_\pm, \Theta_\pm), \breve{V}_+(x_\pm, \Theta_\pm) \in \mathbf{G}_+^{\mathbf{w}}$, $\breve{G}_-(x_\pm, \Theta_\pm), V_-(x_\pm, \Theta_\pm) \in \mathbf{G}_-^{\mathbf{w}}$. The arguments we have used for the maps into $\mathbf{C_1}$ can be applied (with the appropriate modifications due to the restrictions (56)) to this case also because for each $w \in W$ the map ϕ_w^- is holomorphic, so we conclude that the dual σ-model on each cell $\mathbf{C_w}$ is governed by the action (25). Hence, we can rewrite (82) in another two ways:

$$\begin{aligned} F(x_\pm, \Theta_\pm) &= wG(x_\pm, \Theta_\pm)H_-^{-1}(x_+, \Theta_-) \\ &= w\breve{G}(x_\pm, \Theta_\pm)\breve{H}_+^{-1}(x_+, \Theta_-), \end{aligned} \tag{83}$$

where $G, \breve{G} \in \mathbf{C_w}$, $H_- \in \mathbf{G}_-^{\mathbf{w}}$, $\breve{H}_+ \in \mathbf{G}_+^{\mathbf{w}}$. It is clear that the set of equations (76,80,83) defines the map from the total super world-sheet into the double $\mathbf{G}^{\mathbb{C}}$:

$$\begin{aligned} F(x_\pm, \Theta_\pm) &= G(x_\pm, \Theta_\pm)H_-^{-1}(x_+, \Theta_-) \\ &= \breve{G}(x_\pm, \Theta_\pm)\breve{H}_+^{-1}(x_+, \Theta_-), \end{aligned} \tag{84}$$

where $G(x_\pm, \Theta_\pm)$ is the solution of ($\mathbf{G}$)-SWZNW model and $\breve{G}(x_\pm, \Theta_\pm) \in \mathbf{G}$. From this equation we see that $\breve{G}(x_\pm, \Theta_\pm)$ is the classical solution of ($\mathbf{G}, J$)-SWZNW model also because it is obtained from the solution $G(x_\pm, \Theta_\pm)$ by the right multiplication on $\mathbf{G}$-valued function $H \equiv H_-^{-1}\breve{H}_+$ which is satisfied to the equation

$$D_- H = 0. \tag{85}$$

Thus the dual σ-model coincide with the initial one, i.e. N=2 SWZNW model on the real Lie group is PL T-self-dual and PL T-duality transformation is the special type of super Kac-Moody symmetry.

5. Conclusions.

We have shown that Poisson-Lie T-duality in the classical N=2 SWZNW models on the compact Lie groups is governed by the complexifications of these groups endowed with the Semenov-Tian-Shansky simplectic forms, i.e. Heisenberg doubles. Each N=2 SWZNW model admits very natural PL symmetries with respect to the natural actions of the isotropic subgroups forming the complex Heisenberg double.

Under the PL T-duality transformation N=2 SWZNW model maps into itself but this transformation acts as some special super Kac-Moody symmetry on the classical solutions. Thus N=2 SWZNW models on the compact groups are PL T-self-dual.

This result bears a resemblance to the T-self-duality in the self-dual torus compactifications [3]. Note also that in order to apply PL T-duality transformation we used only the left invariant complex structure on the group manifold. Since $(2,2)$ Virasoro superalgebra of symmetries demands also the right invariant complex structure it is an intresting question which certainly deserves futher attention is what happens with the right invariant complex structure under this transformation. Another point which is believed to be intresting is the N=4 SWZNW models generalization of PL T-duality. The N=4 superconformal algebra demands the existing infinite number complex structure on the Lie algebra of the model which are parametrized by the points of two dimensional spheare [23], [28]. It is reasonable to expect the quaternionic geometry will appear in these models.

We expect the PL T-duality exists also in N=2 superconformal Kazama-Suzuki models [36] since these models can be represented as the cosets (N=2 **G**-SWZNW model)/(N=2 **H**-SWZNW model), where **H** is a subgroup from **G** [37].

Another intresting question what is the quantum picture of the PL T-duality in N=2 SWZNW models. Because the Poisson-Lie groups is nothing but a classical limit of the quantum groups [12] there appears an intriguing possibility of a relevance of quantum groups in the T-duality and other superstring applications for example in D-branes [15], [38].

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